# INVERSE HEAT-TRANSFER PROBLEMS IN THE INVESTIGATION OF THERMAL PROCESSES AND THE DESIGN OF ENGINEERING SYSTEMS

O. M. Alifanov

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Formulations, regions of application, and methods of solution of inverse problems of heat transfer are considered for thermal design, simulation, and processing of experimental results

## Formulations and Classifications of Inverse Problems

All the formulations of the heat-transfer problems between a solid or a certain system and the environment in this paper will be examined from the viewpoint of the cause and effect relationship. Among the causal characteristics of the heat-transfer process in a body (system) are the boundary conditions and their parameters, the initial conditions, the thermophysical properties, the internal heat sources and conductivities, and also the geometric characteristics of the body or system. Then any thermal state governed by the temperature field of the object of investigation will be the effect. The build up of cause and effect relations is the aim of direct heat-transfer problems. On the other hand, if it is required to reproduce causal characteristics by means of definite information about the temperature field, then we have some formulation of the inverse problem of heat transfer.

In contrast to the direct problems, formulations of the inverse problems does not correspond to physically realizable phenomena, for instance, it is not possible to reverse the course of the heat-transfer process, and thereby to change the course of time. Therefore, it is possible to speak about the physical incorrectness of the formulation of the inverse problem. Naturally, it already appears as a mathematical incorrectness in the mathematical formalization (most often an instability in the solution), and inverse problems are a typical example of incorrectly formulated problems in heat-transfer theory. The initial formulation of the problem must be predetermined in a special manner in order to obtain a regular solution.

By generalizing problems of this type, we consider all meaningful inverse problems of heat and mass transfer. They can be separated into several large classes. These are primarily the inverse problems of heat conduction (IPHC) when it is assumed that the heat-transport process in a solid is realized purely conductively, or the heat-transfer model in a body is representable by the heat-conduction equation with effective values of the coefficients. In conformity with the causal characteristics introduced above, it is logical to subdivide the inverse problems of heat conduction into boundary, coefficient, retrospective, and geometric inverse problems depending on whether the desired characteristic is among the boundary conditions or the coefficients of the equation, is provisionally reciprocal to the time, or the parameters are determined by the geometric body shape. There may be distinct combined formulations of the IPHC, when causal characteristics of different types enter simultaneously. Depending on the model of the process being used and on the kind of domain of variation of the independent variables, the inverse problems of heat conduction are separated into one- and multidimensional, linear and nonlinear, with fixed or moving boundaries, simply or multiply connected.

It is important to distinguish inverse problems intended to analyze thermal processes and thermal modeling from inverse problems in thermal design, i.e., inverse problems to synthesize engineering systems with required characteristics.

Inverse problems of complex heat transfer, heat transfer in a system of bodies, inverse problems in boundary layer theory, in a conjugate formulation can be introduced analogously to inverse problems of heat conduction. Inverse problems of heat conduction and inverse problems for engineering systems have been studied and applied practically most.

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Let us consider a certain system consisting of n bodies with internal heat liberation (absorption). The bodies of the system exchange thermal energy with the environment and mutually. This heat-transfer process is determined by the parameters of the boundary conditions and the heat-balance equations, by conductive, convective, and radiation type relations, by effective thermophysical characteristics and heat sources, by the geometry and mutual arrangement of the bodies, and also by the initial thermal state of the system. If it is required to compute the running thermal state (temperature mode) by means of the mentioned causal characteristics, then such a computation is the subject of a direct problem of the system heat transfer. In the case when individual causal characteristics are unknown and it is required to determine them by means of known information about the thermal state of the system (actual in a simulation and allowable in design), then inverse problems of the heat transfer of an engineering system are solved. In this case a composite thermal model of the system is used. By applying the method of breaking down the composite model into simple ones, it is possible to go over to formulations of local inverse problems of heat transfer, usually to inverse problems of heat conduction.

## Inverse Problems in Thermal Design

The need to formulate and solve inverse problems of heat transfer appears at all fundamental design stages and the experimental checkout of an engineering system for which the thermal mode is the critical concept and the design characteristics are determined by constraints on the allowable temperature conditions for operation of the individual subsystems, aggregates, units, and elements. Let us briefly examine the appropriate ranges of application of inverse problems.

Making engineering decisions in the design of a certain object is based on optimizing the design parameters with thermal constraints taken into account. Underlying the optimal thermal design is a mathematical thermal model of the system and the target function to be extremized. The model relates the desired design (causal from the viewpoint of formulation of the inverse problem) characteristics to the variables of the state (effect characteristics) and the loading effects, the external and internal heat fluxes, for example. The fully defined heat-transfer and geometric characteristics of the system correspond to possible variants of the design and structural solutions.

Therefore, the problem of optimal thermal design can be considered as the inverse problem of heat and mass transfer in an extremal formulation: Find the required causal characteristics satisfying a state by means of the known conditions governing the allowable thermal state of the system (i.e., in conformity with the given range of variation of the effect characteristics), and also the optimality criterion of the system. The checking of thermal computations in the design stage should refer to the direct problems of heat and mass transfer since the thermal state of a system is sought in this case according to known causal characteristics.

Composite mathematical models of the system are usually constructed by using models of individual elements and of heat-and mass-transfer processes. Experimental methods are hence utilized extensively to select, correct, and verify the consistency of both simple and composite models. Here inverse heat-transfer problems should identify thermal objects (engineering apparatus or heat-and mass-transfer processes) as well as process the results of a thermal experiment and check the appropriate causal characteristic during the tests.

Finally, the object being designed and its composite parts pass through an experimental checkout and test after fabrication of the experimental and operating specimens. Also, inverse heat-transfer problems provide the means for obtaining the results needed, since many experimental-data processing and interpretation processes rely on the solution of these problems.

Let us examine thermal simulation and parameter optimization problems in the thermal design of engineering systems in more detail.

## Thermal Simulation

The general methodology of simulation (identification in the broad and narrow senses [1, 2]) can be made specific in application to thermal simulation whose final purpose is to construct a thermal model of the phenomenon being investigated or the system being designed.

Characteristic of thermal simulation is that in many cases only a passive mode of identifying the object of investigation turns out to be allowable. Excitation effects (heat fluxes, body-surface temperatures, heat-elimination coefficients, etc.) cannot be obtained as a special form of the test signal and are often quantities whose direct measurement is either impossible or fraught with substantial difficulties. In the overwhelming majority of cases the desired causal characteristics cannot also be measured directly. The thermal state of the object as a reaction to an exciting effect can only be obtained in a limited number of points in the space coordinates. These data are aggravated by noise and various errors. Therefore, the mathematical assurance of thermal simulation should rely to a considerable extent on the algorithm for solution of the inverse heat-transfer problems.

Let us separate the thermal-simulation problem into two successive stages: recognition and "training" of the model structure, and intrinsic identification of the model.

To do this, let us represent the mathematical model of the thermal object to be trained (which we shall later understand to be a process or an engineering system) in the provisional operator—vector form:

$$A_M[\bar{\alpha},\ \bar{\beta},\ \bar{T},\ \bar{u},\ \bar{x},\ \tau] = T,$$

where $A_M$ is a nonlinear space—time transformation in the general case, which governs the correspondence between the vectors (vector functions) of the known causal characteristics of the model ($\bar{\alpha}$), the variable causal training characteristics ($\bar{\beta}$), the temperature field $\bar{T}(\bar{\alpha},\ \bar{\beta},\ \bar{u},\ \bar{x},\ \tau)$, and the loading effect $\bar{u}$.

A vector consisting of separate representations of the causal model characteristics is taken as the training-characteristics vector $\bar{\beta}$. It can include individual thermophysical properties, the initial temperature distribution, the boundary conditions or parameters in the boundary conditions, functions and parameters defining the body boundary, etc.

Recognition of the structure of the thermal model will be considered as a problem of recognizing the components of the vector $\bar{\beta}$. The solution of this problem is constructed by means of the results of "yes—no" type answers to checking questions of the following kind: Is the generalized equation of heat conduction adequate for the process under investigation? Is it necessary to consider a two-dimensional rather than a one-dimensional model? Must the displacement of the body boundaries be taken into account? It is necessary to take account of the variability of the thermophysical properties? etc.

The selection and "training" of the identification algorithms (inverse problems) also occurs at this stage. For this purpose such training questions must be used as would permit obtaining answers to the following fundamental methodological questions. What formulations of the inverse problems are most expedient in this case? What methods of solving them would yield the best or sufficient accuracy of the results? What error estimates of the causal characteristics being reproduced can hold depending on the kind and level of the errors in the initial data?

Therefore, the first part of the thermal simulation is a methodological investigation based on the solution of model problems, available experimental results, and specially formulated tests. Consequently, a number of the uncertainties in the selection of the model structure and the identification algorithms is reduced and the accuracy of the characteristics being identified is also estimated prior to the beginning of the second part of the simulation.

On the basis of the results of the first stage, the thermal-model-identification stage assumes the model structure and the optimal algorithms for the solution of inverse problems to be known. The purpose of the second stage is to find the causal characteristics of the model by means of measurement data on real objects by solving the inverse heat-transfer problems. And if we speak about identification of the model of an engineering system, then the purpose is to obtain, on the whole, a possibility of constructing a functional—physical model of the system to be used in subsequent stages of the thermal design or as a check and control of the thermal mode under working conditions.

The algorithmized model of a thermal object is represented as follows in abstract form:

$$A_M^h[\bar{\alpha}_{TPC},\ \bar{T}_0,\ \bar{\Phi},\ \bar{\Gamma},\ \bar{T}_M,\ \bar{u},\ \bar{x},\ \tau] = \bar{T}_M, \tag{1}$$

where $A_M^h$ is the space—time operator approximating $A_M$; for this model it determines the correspondence between the thermophysical characteristics given by the vector $\bar{\alpha}_{TPC}$, the initial distribution (the vector $\bar{T}_0$), the geometric characteristics ($\bar{\Phi}$), the boundary conditions or their parameters ($\bar{\Gamma}$), the discretized temperature field ($\bar{T}_M$), and the loading effect ($\bar{u}$).

The unknown causal characteristics are estimated on the basis of the model (1) and the states of the thermal object are determined.

## Optimal Thermal Design

The selection and determination of the design parameters of heat shield, heat regulation, and thermostat systems are usually considered the principal purpose of thermal design. The general formulation of the appropriate optimization problem is the following: Select a vector of the variable causal characteristics $\bar{p}$ of dimensionality n from some domain P such as to extremize (for definiteness, to minimize, for example), the target function J. The domain of allowable solutions P is extracted by functional-engineering and physical constraints $\{g_i\}_1^l$ which are determined by the temperature mode in the general case.

We therefore have the problem

$$\min_{\bar{p}\in P} J[\bar{p}],$$

$$P = \{\bar{p} : g_i(\bar{p}, \ T(\bar{x}, \tau, \bar{p})) \leqslant 0, \quad i = 1, 2, \ldots, l\},$$

$$T(\bar{x}, \tau, \bar{p}) = A[T(\bar{x}, \tau, \bar{p}), \ \bar{p}]; \ \bar{p} \in R^n,$$

where A is a space—time transformation which sets up a correspondence between the desired vector and the thermal state of the system being designed. As the main design criterion, the target function can characterize the system weight, the coolant or energy consumption, the expenditure on production and servicing, etc.

Let us examine the problem of the optimal design of a heat-shield coating (HSC) in the following sufficiently general formulation in greater detail: Determine the design characteristics of a multilayered coating, one of whose boundary conditions (and the corresponding layer) is subject to external non-steady-state heating, rupture, and entrainment, while the other is subject to cooling by the circulating heat-transfer agent. Let us take the total coating mass as the criterion for the quality of the heat shield.

In many cases the heat-propagation process in an HSC is almost one dimensional. Then the temperature field of the coating along the normal to the surface is representable at a certain point by a system of one-dimensional partial differential equations: The heat-transmission process for the first (k—$l$) layers is described by homogeneous heat-conduction equations

$$C\mathrm{ef}_j(T)\frac{\partial T_j}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda\mathrm{ef}_j(T)\frac{\partial T_j}{\partial x}\right), \quad (x, \tau) \in S_j,$$

$$S_j = \{(x, \tau) : x_{j-1} < x < x_j; \quad x_0 = 0; \quad 0 < \tau \leqslant \tau_m\}, \quad j = 1, 2, \ldots, k-1,$$

and for the last, partially entrained layer by the generalized equation

$$C_{\mathrm{ef}_k}(T, \gamma)\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda_{\mathrm{ef}_k}(T, \gamma)\frac{\partial T}{\partial x}\right) + K_{\mathrm{ef}}(T, \gamma)\frac{\partial T}{\partial x} + Q\mathrm{ef}(T, \gamma),$$

$$(x, \tau) \in S_k = \{(x, \tau) : x_{k-1} < x < X(\tau); \quad 0 < \tau \leqslant \tau_m\},$$

where $\tau_m$ is the maximum value of the HSC operating time, and the subscript ef denotes the effective value of the thermophysical characteristics.

The thermophysical properties of a rupturing layer are known functions of the temperature T and the parameter $\gamma$ which takes account of the dynamics of the change in the properties during heating and rupture of the material. The properties of the remaining layers are given by functions of the temperature.

The motion of the outer boundary of the layer being ruptured, which is determined by the dependence $X(\tau)$, occurs because of linear entrainment of the material and is implicitly related to the external heat-transmission parameters by means of the chemical-kinetics equation and the heat and mass transfer.

The initial condition

$$T_j(x, 0) = \psi_j(x), \quad j = 1, 2, \ldots, k,$$

is known. In the external boundary conditions

$$\lambda_{\mathrm{ef}_1}(T)\frac{\partial T_1(0, \tau)}{\partial x} + q_0(\tau) = 0,$$

$$\lambda ef_k(T)\,\frac{\partial T_k(X(\tau),\ \tau)}{\partial x}+q_k(\tau)=0$$

the conductive heat fluxes $q_0$ and $q_k$ satisfy the following heat-balance equations:

$$q_0(\tau)=\alpha(T_1(0,\ \tau)-T_{cool}),$$
$$q_k(\tau)=q_W(T_W(\tau),\ \tau)-\varepsilon\sigma T_W^4(\tau)-q_{en}(T_W(\tau),\ \tau),$$

where $\alpha$ is the coefficient of heat elimination, $T_{cool}$ is the coolant temperature, $q_W$ and $T_W$ are, respectively, the external heat flux and the temperature of the entrainable HSC boundary, and $q_{en}$ is the heat flux due to the effects of blowing and rupture on the body surface.

The conditions for connection of the layers assume equality of the fluxes and a possible temperature jump at the junctures.

Using this physical model of the object being designed, select the number, material, and thickness of the layers from the condition of a minimum mass of coating. This problem is usually simplified successfully by assuming that the optimal thicknesses in each point r of the coating surface under consideration can be determined independently of the others by the criterion of a minimum specific HSC mass at this point

$$m_r=\sum_{j=1}^{k}\rho_j b_j^r, \tag{2}$$

where $\rho_j$ is the density of the material in the j-th layer and $b_j^r$ is the thickness of the j-th layer at the r-th point of the coating surface.

The optimization problem requires the solution with a number of constraints taken into account, which are dictated by the requirements of the allowable temperature operating conditions for the layers, the specific heat of the coolant, the strength, structural complexity, fabrication technology, and economic expenditures.

We therefore have a combined coefficient-geometric inverse problem in an extremal formulation. Let us separate it into two problems for a practical realization. The problem is solved by the method of sampling for the variation of the thermophysical characteristics in the class of given brands of materials as well as of the number of coating layers. A thickness vector $\bar{b}_r$ of dimension k is sought for each of these variants, which would yield a minimum specific mass for the coating at the given point r and would satisfy the constraints which separate the domain of the allowable temperature mode of operation of the individual layers, i.e., the geometric IPHC with a mass optimality criterion

$$\begin{gathered}\min_{\bar{b}_r\in G} m_r(\bar{b}_r),\\ G\{\bar{b}:g_i(\bar{b},\ T(x,\tau,\ \bar{b}))\leqslant 0;\quad i=1,\ 2,\ \ldots,\ l\},\\ T(x,\ \tau,\ \bar{b})=A[T(x,\ \tau,\ \bar{b}),\ \bar{b}];\quad \bar{b}\in R^k.\end{gathered} \tag{3}$$

is solved by the crux of the matter.

Exactly as in the case of IPHC for the processing of experimental results, inverse problems in the thermal design formulation are incorrectly posed in the general case. This occurs most often because of the possible inconsistency between assignment of the separate conditions for an a priori extraction of the domain of allowable engineering solutions. Hence, not all the constraints should be considered "hard," but the compulsory and desirable among them should be extracted. The preference for one condition over another is determined by the unremovability or irrationality of eliminating some constraint. Other criteria in the design process can be revised and altered.

The strategy of changing the thickness during motion towards the optimum is based on iterative minimization methods. The initial problem is first reformulated into a problem without constraints by using penalty functions.

Let us note one fact in principle. The solution found from the condition of minimum specific mass at a number of characteristic points of the coating can satisfy the constructor of a heat-shield system only when the heat-loading and heat-removal conditions on the boundary surfaces vary comparatively slightly during the passage from one point r to another. In the general case, the vectors $\bar{b}_r^{opt}$ obtained, which are optimal for each point r taken separately, will not agree from the structural and technological viewpoints. This is still another "incorrectness" of the initial formulation of this problem. Because of the change in heat loads on the body surface, a change in layer thickness during passage from one surface point to another can be represented by a sufficiently complex vector function of the coordinates of the HSC support surface. A practical realization of the laws found for the thickness variation turns out to be expedient in connection with structural—technological difficulties.

Therefore, it appears necessary to find a compromise solution which is close to the optimal from the weight viewpoint and satisfies production—economic requirements. Such a solution can be obtained by modifying the initial design criterion because of the addition of a term dictating the necessary degree of smoothness of the thickness-distribution diagram. Assuming for simplicity that profiling of a multilayered heat shield is realized in one coordinate s, we write the new target function in the form

$$\Phi[\bar{b}, \alpha] = \int_0^S \Big[\sum_{j=1}^{k} \rho_i b_j(s)\Big] ds + \sum_{j=1}^{k} \alpha_j \int_0^S [x_j^{(\gamma_j)}(s)]^2 ds, \tag{4}$$

where S is the length of the contour of the HSC reference surface (at x = 0, for example), $\gamma_j$ are the powers of the derivatives, $\alpha_j > 0$. Design practice for HSC states that sufficiently often $x_j(S)$, j = 1, 2, ..., k, are functions which vary slightly and are sufficiently similar in form, and whose smoothness requirements are approximately identical. We shall hence consider that $\alpha_1 = \alpha_2 = \ldots = \alpha_k = \alpha$, $\gamma_1 = \gamma_2 = \ldots = \gamma_k = \gamma$, and, moreover, an adequate degree of smoothness of the desired curves is obtained for $\gamma = 1$ or 2.

Selection of the parameter $\alpha$ as a quantity governing the closeness of the design solution to the optimal is carried out according to the limiting condition on the maximum allowable difference between the total mass of the HSC ($\Delta M_{all}$) obtained as a result of minimizing the functional (4), and the minimum total mass corresponding to the initial criterion (2):

$$|M[\alpha] - M_{min}| - \Delta M_{all} = \min_{\alpha}.$$

The thickness vector function b(S), found by means of the minimum of the functional (4) for the value $\alpha = \alpha_{opt}$, is taken as optimally smoothed.

The optimization method proposed for the HSC parameters, based on regularizing the geometric inverse problem of heat conduction, permits the construction of a fully automated algorithm for heat-shield design.

## General Approach to Obtaining Smooth IPHC Solutions

One of the most general and universal methods of solving incorrectly formulated problems is the Tikhonov regularization method [3, 4]. Let us examine the application of this method to the solution of IPHC for the thermal simulation and interpretation of the results of thermal experiments.

Let us examine the domain

$$D = \{(x, \tau);\ X_1(\tau) < x < X_2(\tau);\quad 0 < \tau \leqslant \tau_m\},$$

in which is given the nonlinear heat-conduction equation

$$C(T)\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right) + K(T)\frac{\partial T}{\partial x} + Q(T) \tag{5}$$

in the plane (x, $\tau$). The boundary-value problem for (5) assumes the following initial temperature distribution

$$T(x, 0) = \psi(x), \quad X_1(0) \leqslant x \leqslant X_2(0) \tag{6}$$

and the boundary conditions

$$L_i(T) = \varphi_i(\tau), \quad i = 1, 2, \tag{7}$$

to be given, where the operators can correspond to boundary conditions I and II of the boundary-value problems, i.e.,

$$L_i(T) \equiv T(X_i(\tau), \tau)$$

or

$$L_i(T) \equiv -\lambda(T(X_i(\tau), \tau)) \frac{\partial T(X_i(\tau), \tau)}{\partial x},$$

and a mixed boundary-value formulation is also possible.

The direct problem is to seek the function $T(x, \tau)$ satisfying (5) in the open domain D which satisfies conditions (6) and (7) and is continuous together with the gradient $\partial T(x, \tau)/\partial x$ in the closed domain $\bar{D}$.

If one of the functions $\varphi_i(\tau)$, $\psi(x)$, $C(T)$, $\lambda(T)$, $K(T)$, $Q(T)$ is not known and this function [we denote it by $u(y)$] and the temperature field $T(x, \tau)$ must be known by means of the known remaining and additional conditions $T(x^*, \tau) = f(\tau)$, or $T(x, \tau^*) = \bar{f}(x)$, where $x^*$, $\tau^*$ are given points or curves [$x^*(\tau)$, $\tau^*(x)$] within the domain D, then we have the inverse problem of heat conduction. Henceforth the additional condition will be written in the generalized form $T(\zeta, \zeta^*) = f(\xi)$. Let us assume that the solution of this problem exists and is unique; however, the stability condition is spoiled.

If the extremal formulation of the IPHC is formulated as a problem to search for an element u from some domain U governed by previously known physical constraints [the positivity of the functions $C(T)$, $\lambda(T)$, and $X(\tau)$, for example], which realizes the minimum of the rms deviation between the design and given temperatures

$$\rho(u) = \|T(\zeta, \xi^*, u(y)) - f(\xi)\|^2_{L_2},$$

then such a problem turns out to be incorrectly formulated. Let us regularize this problem by modifying it as follows:

$$\min_{u \in U} [\rho(u) + \alpha\Omega(u)], \tag{8}$$

where $\Omega(u)$ is the Tikhonov regularizer and $\alpha > 0$.

As experience with the solution of incorrect inverse problems shows [5, 6], in many cases the presence of a priori information about the smoothness of the solution permits the assumption

$$\Omega(u) = \|u - u^*\|^2_{W_2^n},$$

where $n = 1$ or 2, and $u^*$ is a trial solution.

The extremal of the functional (8), found for a specific agreement between the parameter $\alpha$ and the error of the entrance data, will yield an approximate solution of the IPHC.

The iterative minimization method (8) results in the convergent sequence

$$u_{j+1} = u_j + \Delta u_j, \quad j = 0, 1, \ldots,$$

where $u_0$ is the initial approximation.

The increments $\Delta u_j$ for the passage from iteration to iteration are determined by the expression

$$\Delta u_j = -\beta G \quad [u_j],$$

where G is a vector indicating the direction from the point $u_j$ and $\beta$ is the magnitude of the step along this direction.

The gradient of the minimizing functional must be known in methods of gradient type for the determination of G. Depending on the kind and complexity of the problem to be solved, it can be calculated by three methods: analytically, by using the adjoint boundary-value problem [7, 8], and experimentally.

Good practical results [5, 6] on the selection of the parameter $\alpha$ yield known principles of a residual [9] which regularizes the functional [10, 11], or their combination with the method of a quasioptimal parameter [12].

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# IMPULSE-FUNCTION METHOD FOR HEAT-TRANSFER DYNAMICS IN A CHANNEL

B. P. Korol'kov and É. A. Tairov

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A method of solving the boundary problem for heat-transfer dynamics in a channel is proposed; the problem is reduced to integral equations of Volterra type.

Nonsteady one-dimensional motion of heat carrier in a heated channel is considered. The problem is to determine the change in the parameters (temperature, flow rate, pressure) due to perturbation of the external conditions. The change in the flow rate and pressure at the channel inlet are related by the boundary conditions for the equation of motion. In most of the known works, the conditions at the right-hand boundary were either completely disregarded [1], or else were assumed to affect only the pressure deviation at the inlet and the flow rate was assumed to be given [2]. If conditions are specified at both boundaries, it is necessary to solve a boundary problem for the system of equations describing the heat transfer and hydrodynamics. A more complete formulation of the problem is possible if numerical methods are used for the direct integration of the differential equations, but to date this approach has been used mainly in the context of scientific research because the computational algorithms are too complex for use in engineering practice.

The present work describes a method by which, in the linear case, the boundary problem can be reduced to two integral Volterra equations of the second kind of convolution type; analytic expressions are obtained for the impulse function relating the changes in input and output parameters. Computer solution of the integral equations is straightforward.

Taking the equations of statics into account [3], a linearized system of conservation equations may be written for the parameter deviations:

$$\frac{\partial \Delta D}{\partial z} + f\frac{\partial \Delta \rho}{\partial \tau} = 0, \tag{1}$$

$$D_0\frac{\partial \Delta i}{\partial z} + f\rho_0\frac{\partial \Delta i}{\partial \tau} + \frac{\partial i_0}{\partial z}\Delta D = \Delta\alpha h(\theta_0 - t_0) + \alpha_0 h(\Delta\theta - \Delta t), \tag{2}$$

$$\Delta q - g_w c_w\frac{\partial \Delta \theta}{\partial \tau} = \Delta\alpha h(\theta_0 - t_0) + \alpha_0 h(\Delta\theta - \Delta t), \tag{3}$$

$$\Delta p_1 - \Delta p = \frac{2\delta p_0}{D_0}\Delta D - \frac{\delta p_0}{\rho_0}\Delta\rho,$$

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